Heat loss and temperature profile along an insulated pipe simulated with Mathematica 9 and FloEFD 11.

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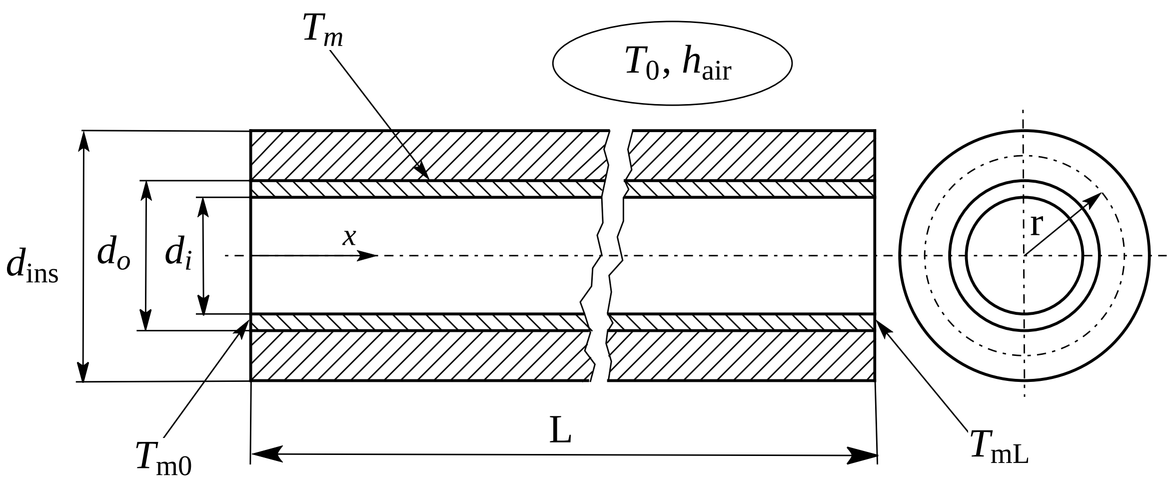
Introduction

In this article we propose to analyze the heat rate and temperature distribution in a pipe insulated with a thin insulation layer. The pipe with inner diameter d_i, outer diameter d_o and the length L has the temperature T_{m0} and T_{mL}at the ends. The insulation of the pipe has outer diameter d_{ins} and thermal conductivity k_{ins}. Insulation exchanges heat by convective heat transfer with the environment at temperature T_0 and the heat trasnfer coefficient is h_{air} . This practical problem, which is illustrated graphically in the figure below, may be encountered e.g. in the case of two hot casing components rigidized with a pipe which is insulated for personnel protection.



Import modules and create special symbols - details hidden

Input Data

T_{air} = 20. ℃
T_{m0} = 430. ℃
T_{mL} = 430. ℃
h_{air} = 10. W/m^{2}K
k_m = 20. W/m·K
k_{ins} = 0.04 W/m·K
d_i = 0.06 m
d_o = 0.07 m
d_{ins} = 0.13 m
L = 1. m

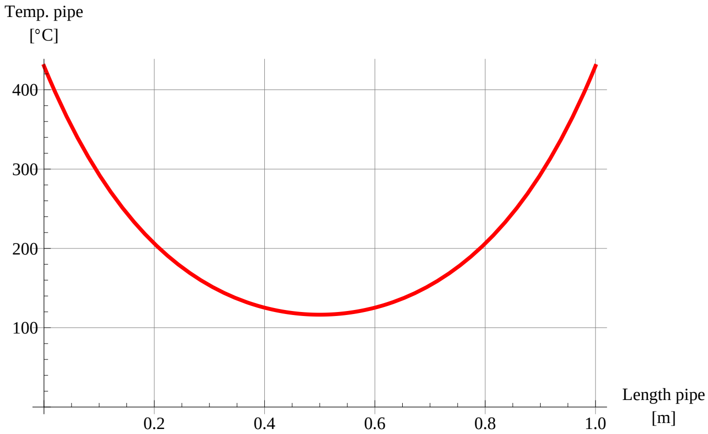
The global heat transfer coefficient, referred to insulation external area at d_{ins}, between the pipe and the insulation is calculated by the formula :

$$U_{ins} = \frac{1.}{\frac{\ln\left(\frac{d_{ins}}{d_o}\right)}{2.0\,\frac{k_{ins}}{d_{ins}}} + \frac{1.0}{h_{air}}}$$

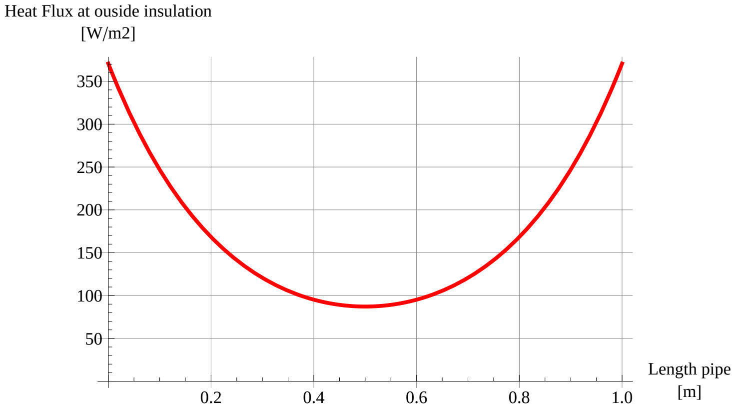
Parameter $P = \pi \frac{U_{ins}}{A_{cm}} \frac{d_{ins}}{k_m}$, adimensional, is used in the formulae below.

The temperature of the pipe, T_m, is assumed constant in its cross section due to the high value of k_m which is 500 times lager than k_{ins}.
Below are given the graphs obtained for the input above using the theoretical considerations given below using *Mathematica* 9 CAS software.

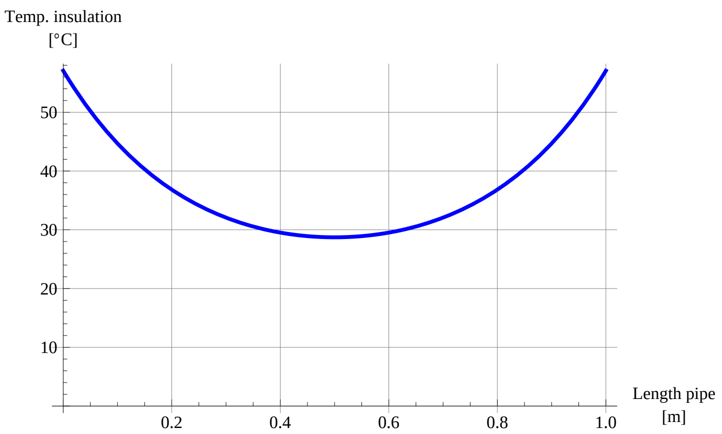
The temperature of the pipe:



Heat flux on insulation external surface: $q(x) = U_{ins}(T_m(x) - T_{air})$:



Temperature on insulation external surface: $T_{ins}(x) = \frac{U_{ins}(T_m(x) - T_{air})}{h_{air}} + T_{air}$:

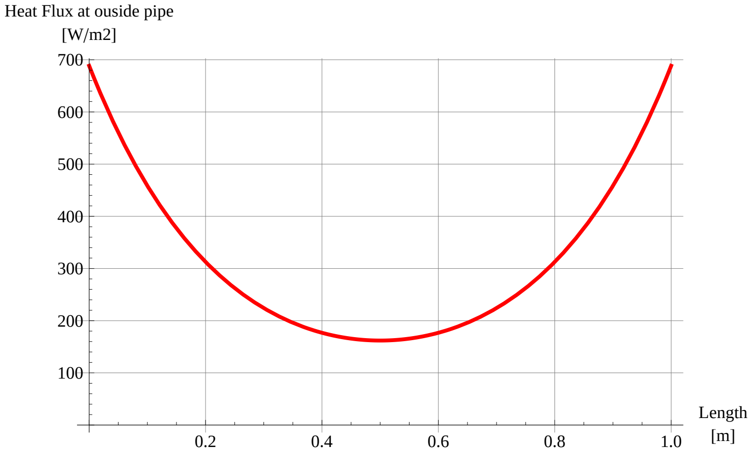


Teoretical considerations

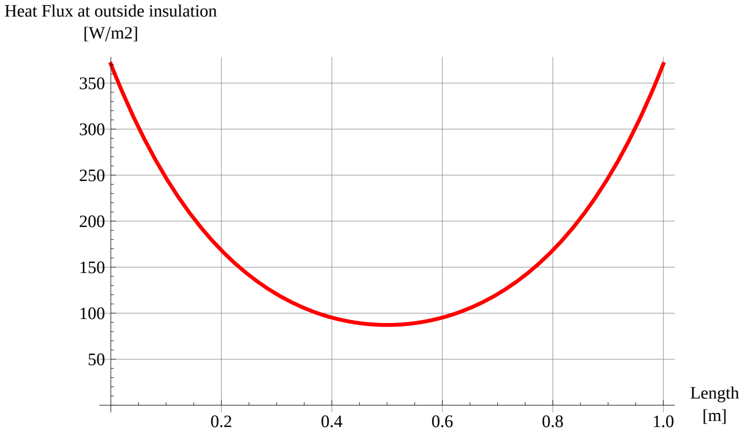
The problem is a one-dimensional problem in the direction *x* along the center line of the pipe; we take *x=0* at the left end of the pipe.
We disregard the conduction in the *x*-direction in the insulation and the conduction in the pipe perpendicular on the *x*-dimension (see below).

- Heat losses dQ_{loss}/dx

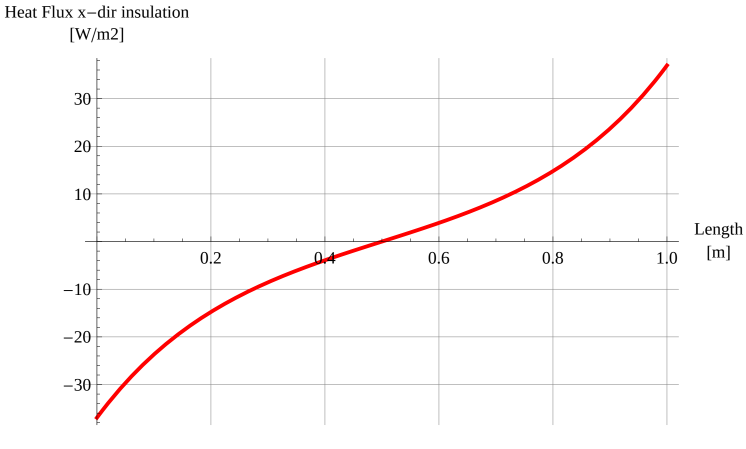
The justification for disregarding the conduction in the x-direction for the example studied above is as follows:
We compare the heat flux in the *r*-direction with that in the *x*-direction.
The heat flux in the *r*-direction is:
 - at the surface of the pipe, d_o:



 - and at the surface of the insulation, d_{ins}:



On the other hand, the heat flux through insulation in the *x*-direction is approximatelly:



Conclusion: the x-direction heat flux through insulation is one order of magnitude smaller and, within reasonable approximation, dQ_{loss} travels only perpendicular on the x-direction and is constant in any cylindrical cross section.

Integrate $dQ_{loss}/dx = 2\pi\, r\, k_{ins}\, dT/dr$ from $r_o = d_o/2$ to $r_{ins} = d_{ins}/2$ to get T_m - $T_{ins} = \frac{dQ_{loss}/dx}{2\,\pi\,k_{ins}} \ln\left(\frac{d_{ins}}{d_o}\right)$.
Where T_m is the temperature of the pipe and T_{ins} - temperature on insulation external surface.

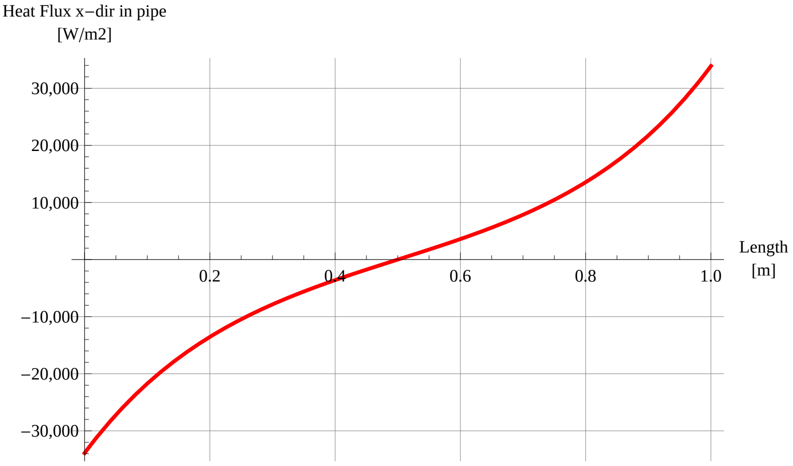
At the otside diameter of the insulation, d_{ins} we get T_{ins} - $T_{air} = \frac{dQ_{loss}/dx}{h_{air}\,\pi\,d_{ins}}$.

Summing up these two equations we get $dQ_{loss}/dx = U_{ins}\,(T_m - T_{air})\,\pi\,d_{ins}$, $\pi\,d_{ins}\,dx$ being the area of external insulation of the dx element.

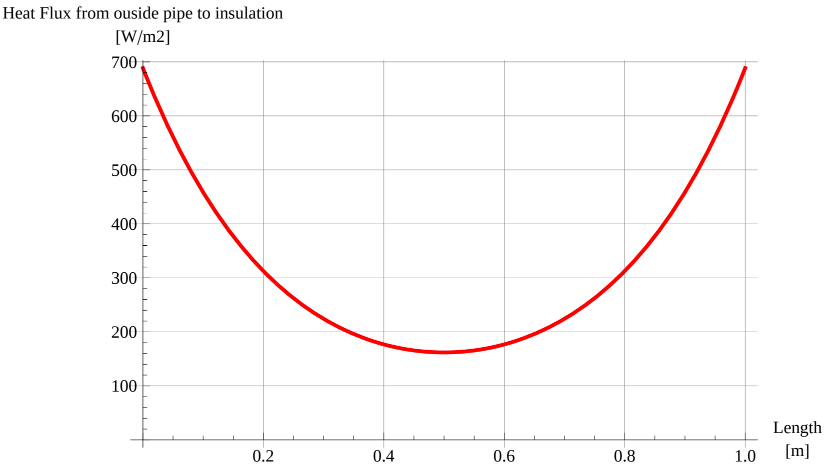
This dQ_{loss}/dx is considered below as being the heat loss [W/m] from the dx element of metallic pipe.

- Differential equation of convective heat transfer with heat loss in the dx element of the metallic pipe

We disregard the conduction in the pipe perpendicular on the x-direction (see below). The only relevant heat flux is in the x-direction through the pipe:



On the other hand, the heat flux from the pipe d_o to the insulation in the r-direction (the heat sink) is approximatelly 2% of the flux in x-direction:



We use the known differential equation $k_m\, T_m{}''[x]$ = - Heat Source (or Heat Gain).
We have a Heat Sink (or Heat Loss), and the differential equation becomes:

$$\begin{cases} T_m''(x) == P\,(T_m(x) - T_{air}) \\ T_m(0) == T_{m0},\ T_m(L) == T_{mL} \end{cases}$$

which is solved with the function DSolve[], see the output above, to calculate the pipe metal temperature $T_m(x)$.
Not all details are worked out above and the reader is encouraged to work it out in detail, to be able to develop similar calculation methods.

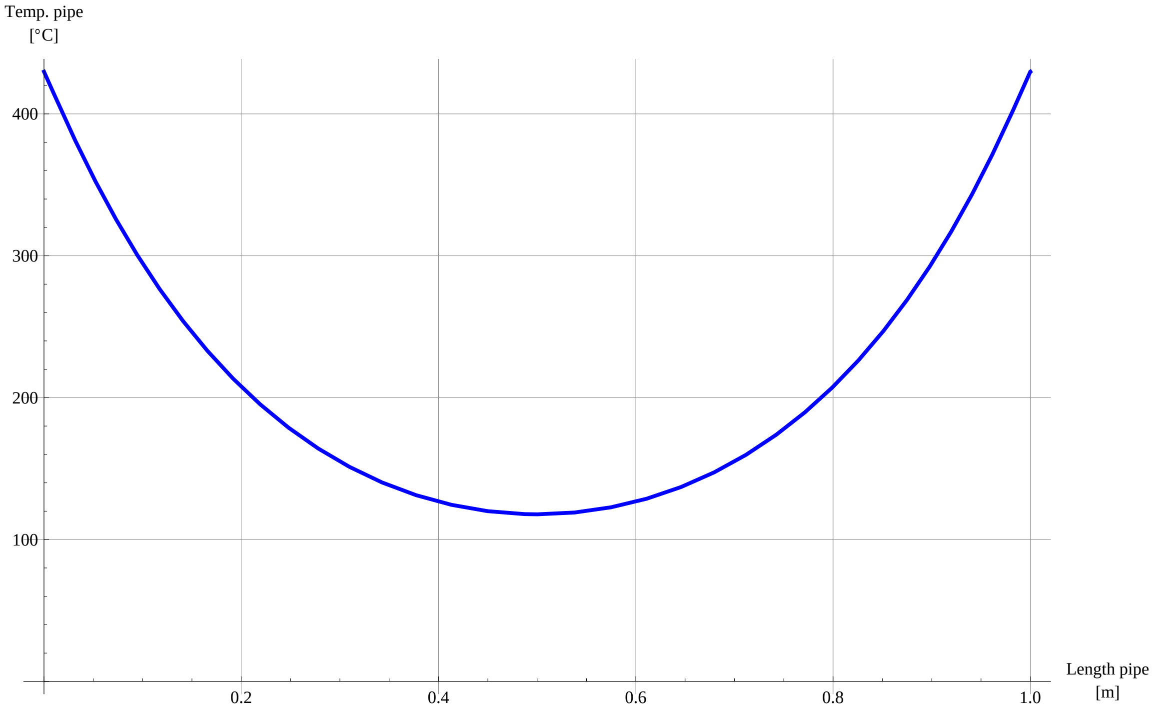
Comparison with CFD simulations

We made a CFD analisys of the same problem with FloEFD software. These calcualtion are based on full 3D conduction in both the metalic and in the insulation. The CFD results of pipe temperature at the outer diameter are presented below:

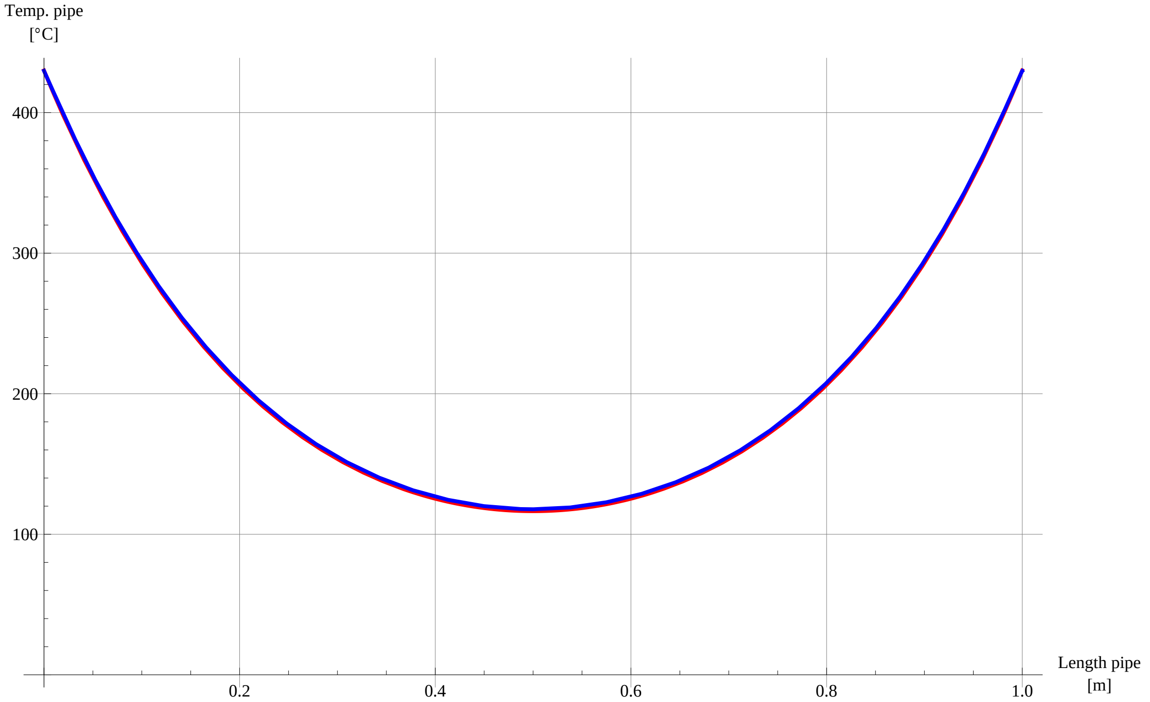
```
CFDlst = Import["Desktop/InsulPipe/CFDresults.xlsx", {"Data", 2}][[5 ;; All]];
```

x [m]	T [℃]	x [m]	T [℃]	x [m]	T [℃]	x [m]	T [℃]
0.	429.76	0.219126	195.35	0.500301	117.799	0.825679	226.261
0.00952389	414.957	0.247995	178.841	0.537889	119.058	0.850939	246.882
0.0317772	381.095	0.278067	164.162	0.574878	122.747	0.874997	268.928
0.0522262	352.401	0.309643	151.261	0.610964	128.774	0.897852	292.258
0.0729759	325.769	0.342722	140.257	0.645848	137.012	0.919804	317.065
0.0946277	300.483	0.377305	131.272	0.679529	147.386	0.940855	343.25
0.117182	276.64	0.413091	124.484	0.711706	159.714	0.961304	371.108
0.140939	254.052	0.449778	120.015	0.742379	173.874	0.982354	402.324
0.165598	233.104	0.487068	117.946	0.771549	189.742	0.999947	429.738
0.19176	213.413	0.499999	117.799	0.799215	207.182	1.	429.76

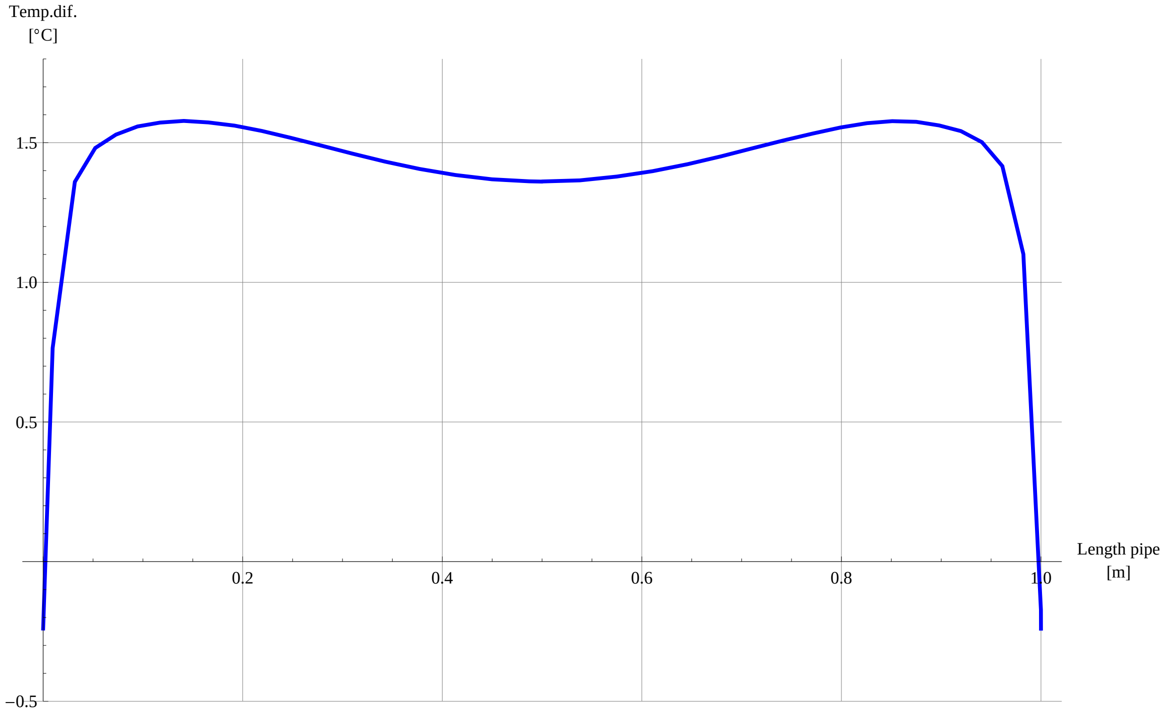
The temperature of pipe at the outer diameter in x – direction from CFD results is :



The temperature of pipe from calculation with assumption of constant temperature in cross section and no heat transfer in insulation in x - direction (red line) and from CFD results (blue line) are as below :



The difference between CFD and mathematical results is small and is difficult to observe in previous graph. The next graph shown only this difference along the pipe.



■ Conclusion :

The CFD results confirms that conduction heat transfer in transversal direction in pipe and insulation is negligible and the mathematical model has a relative error less than 2% as compared with the CFD results.
Please note that for thick insulation the assumption of no heat transfer in insulation in *x*-direction may not be valid and the error will be higher.

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